

Discovering the Fundamental Theorem of Calculus

Objective: Given a function $y = f(t)$, find an “area function” $A(x)$, defined to be $A(x) = \int_a^x f(t)dt$ which represents the area “under” f from $t=a$ to $t=x$.

Instructions: Complete steps a through d below for the given functions and intervals.

- Make a sketch of the function $y = f(t)$ and shade the region on the interval given.
- Use a geometric formula to find the area of the shaded region. Your answer should be a function of x and, in fact, be $A(x) = \int_a^x f(t)dt$. If necessary, simplify your answer by multiplying the terms.

c. Check your answer (function) by graphing. Enter into Y1, $Y1=falnt(F(T), T, a, x)$ and into Y2, your answer for $A(x)$. If the two graphs are not the same curve, find your mistake and regraph.

d. Record your answer on this worksheet.

Examples: 1. $f(t) = 5$ on $[0, x]$. $A(x) = \int_0^x 5 dt = \underline{\hspace{2cm}}$

2. $f(t) = t$ on $[2, x]$. $A(x) = \int_2^x t dt = \underline{\hspace{2cm}}$

Problems: 3. $f(t) = 3$ on $[-1, x]$. $A(x) = \int_{-1}^x 3 dt = \underline{\hspace{2cm}}$

4. $f(t) = t + 3$ on $[0, x]$. $A(x) = \int_0^x (t + 3) dt = \underline{\hspace{2cm}}$

5. $f(t) = 4t$ on $[0, x]$. $A(x) = \int_0^x 4t dt = \underline{\hspace{2cm}}$

6. $f(t) = 4t$ on $[1, x]$. $A(x) = \int_1^x 4t dt = \underline{\hspace{2cm}}$

7. $f(t) = 2t + 3$ on $[0, x]$. $A(x) = \int_0^x (2t + 3) dt = \underline{\hspace{2cm}}$

8. $f(t) = 2t + 3$ on $[-1, x]$. $A(x) = \int_{-1}^x (2t + 3) dt = \underline{\hspace{2cm}}$

9. $f(t) = -3t + 7$ on $[-2, x]$. $A(x) = \int_{-2}^x (-3t + 7) dt = \underline{\hspace{2cm}}$

10. $f(t) = -3t + 7$ on $[1, x]$. $A(x) = \int_1^x (-3t + 7) dt = \underline{\hspace{2cm}}$

Conjecture: What is the relationship between the “area function” $A(x)$ and the original function $f(t)$?
